Noise reduction and signal to noise ratio improvement in magneto-optical polarization rotation measurement

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Received April 24, 2018; accepted June 6, 2018; posted online July 30, 2018

The measurement of an extremely small magneto-optical polarization rotation angle with high sensitivity is integral to many scientific and technological applications. In this Letter, we have presented a technique based on Faraday modulation combined with the optical differential method to measure an extremely small polarization rotation angle with high sensitivity. The theoretical and experimental results show that common mode noise is reduced appreciably and signal to noise ratio is enhanced. The effectiveness of this technique has been demonstrated by measuring the Verdet constant of terbium gallium garnet glass and measuring the small polarization rotation angle. A sensitivity of enhancement of one order of magnitude has been achieved using differential detection based on Faraday modulation.

OCIS codes: 120.5410, 000.3110, 040.1880, 120.5475.

doi: 10.3788/COL201816.081201.

The optical polarimetry based on Faraday rotation finds many scientific and technological applications, in which ultrasensitive measurement of an extremely small polarization rotation angle is of primary importance[1–4]. The Faraday rotation, a well-known magneto-optical effect, refers to the rotation of plane polarized light under the action of the applied axial magnetic field. The typical applications involving the measurement of Faraday rotation include highly sensitive atomic magnetometers[1,3,5], atomic sensors[6,7], experiments on fundamental physics[2,4], optical isolators[8], optical modulators[9], magnetic field, and current sensor[10–12]. Different polarimetry techniques have been demonstrated to measure the Faraday rotation with and without any modulation[2,4,12–20]. The simplest polarimeter requires two linear polarizers, a laser light source, and a photodetector (PD). The first polarizer sets the initial polarization state of the laser light while the second polarizer and detector determines the final polarization state after interaction with the sample. The rotation angle is determined by using Malus law[19]. For a polarimeter without involving any modulation, the sensitivity is generally limited by the extinction ratio of polarizers used[14]. The extinction ratio of typical birefringent polarizers is 1×10^{-6}. Although a lower extinction ratio (1×10^{-9}) can be obtained by using a harmonic generating crystal[21], it is not generally a convenient approach.

Another commonly used technique is balanced detection, in which a Wollaston prism or polarizing beam splitter is employed to divide the output from the analyzer into two orthogonal linearly polarized beams[12]. In this technique, the intensities of the two orthogonal beams are detected separately, and the rotation is deduced after manipulating the two input signals independent of the intensity of the incident light. This technique is equally useful for both AC and DC Faraday rotation measurements.

The use of modulation techniques allows one to measure the Faraday rotation with enhanced sensitivity by suppressing the noises and without requiring polarizers with the highest extinction ratio[14]. The desired signal is extracted at a particular modulation frequency, while signals at other frequencies are suppressed electronically. A widely used modulation technique by means of magneto-optical effects is known as Faraday modulation. In this technique, the plane of polarization of light is modulated by the optical Faraday effect[1–4,13,15,17,22]. The spectral decomposition of the Faraday rotation output signal gives a steady state DC component and modulated AC components. The information of Faraday rotation is deduced by some electronic processing and filtering circuits. The modulation technique using two crossed polarizers is characterized by low average laser power. In balanced polarimetry, on the other hand, the polarizers are set at a large angle of normally 45°, and the DC component is much larger than that in the case of modulation technique. Thus, the modulation technique is advantageous in terms of reducing shot noise and laser intensity noise. However, Faraday polarimetry suffers from laser intensity fluctuation and thermal noise, and this limits the sensitivity of the measurement[22].

In this Letter, we have presented a method to measure an extremely small polarization rotation employing both optical noise canceling and modulation technique. The proposed measurement technique has been verified by measuring the Verdet constant of terbium gallium garnet (TGG) and small polarization rotation angle measurements.

The principle of measuring polarization rotation based on Faraday modulation is depicted in Fig. 1. A Faraday modulator and a rotation generating cell are placed between two crossed polarizers. The laser beam is modulated by a small angle θ_m due to the Faraday effect. The beam

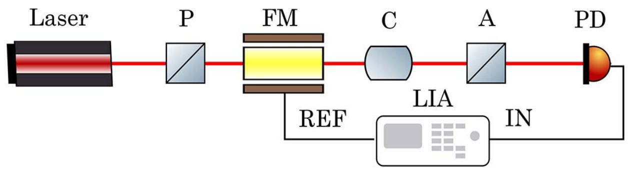

Fig. 1. Polarization rotation measurement system using Faraday modulator. P, polarizer; FM, Faraday modulator; C, rotation generating cell; A, analyzer; PD, photodetector; LIA, lock-in amplifier.

then travels through the rotation generating cell and experiences an additional optical rotation of angle φ. The first harmonics of the signal detected by a PD is extracted by a lock-in amplifier referenced at the excitation frequency. The acquired first harmonic component signal can be converted from time domain to frequency domain by fast Fourier transform (FFT).

The detected signal can be modeled using normalized Jone vectors and matrices as

$$E_{\mathrm{PD}} = M_A \times M_C \times M_F \times M_P \times M_i, \tag{1}$$

where M_A, M_C, M_F, M_P, M_i are the Jones matrix of the analyzer, rotation generating cell, Faraday modulator, polarizer, and polarization state incident light, respectively. The Jones vector of the incident laser in terms of electric field amplitude E, such as detector intensity, is given by

$$M_i = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{2}$$

The Jones matrix of the crossed polarizer can be written as

$$M_P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \tag{3}$$

The rotation generating cell and Faraday modulator can be represented by the matrices as

$$M_C = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}, \tag{4}$$

$$M_F = \begin{bmatrix} \cos(\theta_m \sin\omega t) & -\sin(\theta_m \sin\omega t) \\ \sin(\theta_m \sin\omega t) & \cos(\theta_m \sin\omega t) \end{bmatrix}. \tag{5}$$

The Jones matrix of the analyzer is given by

$$M_A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{6}$$

Using Eqs. (2)–(6), the intensity of the detected signal is

$$I_{\mathrm{PD}} = E_{\mathrm{PD}}^2 = I_0 \sin^2(\varphi + \theta_m \sin\omega t), \tag{7}$$

where I_0 is the intensity of incident laser, φ is the polarization rotation angle to be measured, and θ_m is the depth of the modulation. Using small angle approximations ($\theta_m < 1$, $\varphi \ll 1$), Eq. (7) can be expanded into[23]

$$I(t) \approx I_0\left(\frac{\theta_m^2}{2} + 2\varphi\theta_m \sin\omega t - \frac{\theta_m^2}{2}\cos 2\omega t\right). \tag{8}$$

From Eq. (8), the detected signal is comprised of the DC term, the signal at the modulation frequency, and the term involving the second order of modulation frequency. It is evident that the DC component is proportional to laser intensity and the main cause of the shot noise. The first harmonics are proportional to the polarization rotation angle to be measured. The undesirable second harmonics are nearly independent of φ. The first harmonic extracted by lock-in amplifier referenced modulation frequency is given as

$$I_\omega = 2I_0\varphi\theta_m. \tag{9}$$

In the above analysis, it has been considered that all optical elements are free from defects and are perfectly aligned. However, in reality, it is found that polarizers will deviate from orthogonality due to mechanical drift of the experiment; in addition to this, the non-ideality of optics and laser intensity fluctuation will introduce noise in the detected signal. Consequently, the first harmonics of the detected signal will be given by

$$I_\omega = 2I_0\varphi\theta_m + \mathrm{Noise}. \tag{10}$$

Equation (9) discloses that the first harmonic signal strength is dependent on the original light intensity I_0 and modulation depth θ_m. The modulation depth of the Faraday modulator is given by

$$\theta_m = VBL, \tag{11}$$

where V is the Verdet constant, B is the magnetic field of the solenoid coil of the modulator, and L is the length of the magneto-optical glass. It is also evident from Eq. (9) that I_ω and signal strength can be increased by increasing θ_m. However, the magnitude of undesirable second harmonics is also increased considerably with increasing θ_m. A large driving current of the Faraday modulator is required to achieve an optimal modulation angle, which, in turn, generates heat that is not desirable, especially for paramagnetic glass with a strong temperature dependent Verdet constant. The nonhomogeneity of the axial magnetic field of the driver coil is another disturbing factor. The effect of thermal noise and the nonhomogeneity of the axial field on the stability of θ_m can be mitigated by using a solenoid coil optimized with respect to homogeneity over the region of interest and power dissipation per degree of rotation. The procedure on the design of such coils has been reported in our previous work[24]. The above analysis shows that the small polarization rotation signal will be isolated from low-frequency noise by means of the modulation using single beam detection technique.

However, the laser intensity fluctuation and other noises associated with the imperfection of optical elements and misalignments of the optical system and effect of second harmonics cannot be suppressed in the single beam detection technique.

These problems associated with the single beam detection technique can be overcome by exploiting the advantage of differential detection based on the fact that the useful signal and noise are carried in a different way by the beam of light; the signal is a modulation of the direction of polarization of the light, while the noise is essentially an amplitude modulation. Therefore, the modulated beam can be split into two beams, in which the noise appears in common mode, so that the noise could be canceled out by detecting the two beams by a differential detection, as shown in Fig. 2. The modulated beam before the measuring cell is converted into two identical beams using a non-polarizing beam splitter (NPBS). The transmitted beam passing through the cell and crossed analyzer A1 is detected by PD1, and it is the usual single detection arrangement as discussed earlier. The reflected part of the modulated beam after passing through the crossed polarizer A2 is detected by PD2 and acts as a reference beam. The intensities of the two beams detected by the two PDs are

$$I_1 = \frac{I_0}{2}\sin^2(\varphi + \theta_m \sin \omega t) + \text{c.m.n}, \quad (12)$$

$$I_2 = \frac{I_0}{2}\sin^2(\theta_m \sin \omega t) + \text{c.m.n}, \quad (13)$$

where I_1 and I_2 are intensities of PD1 and PD2, respectively, and c.m.n is common mode noise.

The spectral decomposition of the detected signals of PD1 and PD2 can be written by expanding Eqs. (12) and (13) as

$$I_1 \approx I_0\left(\frac{\theta_m^2}{4} + \varphi\theta_m \sin \omega t - \frac{\theta_m^2}{4}\cos 2\omega t\right) + \text{c.m.n}, \quad (14)$$

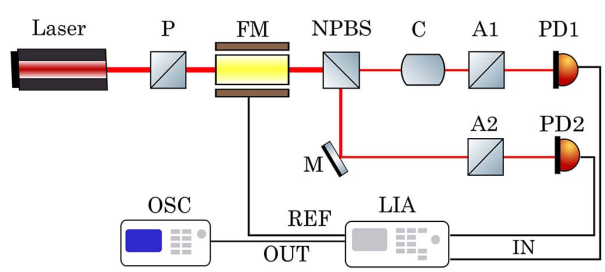

Fig. 2. Schematic of the Faraday modulation-based polarization rotation measurement setup using differential detection. P, polarizer; FM, Faraday modulator; NPBS, non-polarizing beam splitter; M, mirror; C, rotation generating cell; A1, A2, analyzers; PD1, PD2, photodetectors; LIA, lock-in amplifier; OSC, oscilloscope.

$$I_2 \approx I_0\left(\frac{\theta_m^2}{4} - \frac{\theta_m^2}{4}\cos 2\omega t\right) + \text{c.m.n.} \quad (15)$$

It can be seen from Eq. (15) that the signal of PD2 contains only the DC term and second harmonic. The outputs of two PDs given by Eqs. (14) and (15) can be combined differentially. Assuming that the gain of both the detectors is same, the first harmonics extracted by the lock-in amplifier referenced at the modulation frequency are given by

$$S = \eta I_0 \varphi \theta_m, \quad (16)$$

where η is a factor representing the photoelectric efficiency and gain of the preamplifier circuit. As the two beams come from the same light source, all of the fluctuations that arise from light intensity, the imperfection of optical components, mechanical drifts of the experiment, and effect of second harmonic will be canceled out in the ideal case. Furthermore, an investigation on a qualitative comparison of the signal to noise ratio (SNR) suggests that there is a considerable increase in SNR in the case of the technique employing modulation and differential detection together compared with the single beam detection technique.

The schematic of the measurement system based on differential Faraday modulation to detect an extremely small polarization rotation is shown in Fig. 2. The total measuring system consists of a laser, a Faraday modulator, two PDs, a lock-in amplifier, function generators, a power amplifier, a rotation generating cell and optical elements. The rotation generating cell consists of a small coil wrapped on TGG glass to generate the small rotation angle. The Faraday modulator includes a magneto-optic glass surrounded by dense coil driven by a power amplifier working at the modulation frequency of a few kilohertz (kHz). After passing through the Faraday modulator, in which the magnetic field oscillates at a frequency of a few kHz, the laser beam is modulated by a small angle due to the Faraday effect. The modulated beam before the rotation generating cell is converted into two identical beams by an NPBS. The transmitted beam passing through the rotation generating cell experiences an additional optical rotation angle, and it is detected by PD1 after passing through crossed analyzer A1. This arrangement alone forms the conventional single beam measurement technique based on Faraday modulation. While in case of differential detection, the reflected part of the modulated beam passing through crossed polarizer A2 and detected by PD2 acts as a reference beam. The outputs of the two detectors are fed at two channels of a SR830 lock-in amplifier set in A-B (differential) mode. The first harmonics of the detected differential signal are extracted by the lock-in amplifier referenced at the modulation frequency. The acquired first harmonic component signal is converted from time domain to frequency domain by FFT.

A solenoid coil of 90 mm length was used for the Faraday modulator. The variation of the axial field of the coil was measured using a sensitive Gaussmeter with the step of 5 mm along the long axis of the coil, and it is shown in Fig. 3. It is evident that the magnetic field is fairly uniform over the middle region of the coil length, and there is significant variation at the ends of the coil. In our case, the length of the magneto-optic glass was 30 mm. The maximum variation in magnetic field over this length is 3%. Since Faraday rotation is proportional to the magnetic field, the relationship between the peak field and the current was confirmed by measuring the magnetic field as a function of the solenoid driving current. The linear relationship between the magnetic field and driving current resulted in magnetic field generation of 112 G/A (rms).

In order to examine the performance of the setup in the micro degree range of rotation measurements, we have measured Faraday rotation and, hence, Verdet constant V of TGG glass under a small AC magnetic field at 852 nm. Fig. 4 shows the set of data of rms rotation as a function of the numerical integrated field over the length of the sample. The value of V measured using this method was -3.352×10^{-3} deg /(G · cm), which is in excellent agreement with the value deduced from the Verdet constant dispersion relation and the fitting parameters of the experimentally measured values curve at other wavelengths, as reported in the literature[25].

The sensitivity of the two measurement systems was evaluated with a test angle of 1.13×10^{-5} rad generated by the rotation generating cell, which was excited by a voltage of $5 \times 10^{-3}\,V_{\text{P-P}}$ at 30 Hz. The signal response was maximized by appropriate experimental conditions. Typical noise spectra are shown in Fig. 5 for quantitative analysis of the sensitivity of the two measurement techniques. The common mode noise mainly consists of laser intensity fluctuations and noises associated with the imperfection of optical elements, misalignment of the optical system, and effects of second harmonics. For the single beam detection, common mode noise cannot be canceled out, while this kind of noise can be eliminated by differential detection. It is evident from Fig. 5 that with differential detection this kind of noise was suppressed effectively, and the sensitivity of the measurement was improved. It was found that sensitivity was increased from 3.66×10^{-7} rad · Hz$^{-1/2}$ for single beam detection to 3.43×10^{-8} rad · Hz$^{-1/2}$ for the differential detection, which represents about one order of magnitude improvement in sensitivity in the measurement of small polarization rotation angles.

In conclusion, we have demonstrated a technique to measure an extremely small polarization rotation angle with high sensitivity based on Faraday modulation combined with the optical differential method. The signal strength can be increased with modulation depth without disturbing the effect of the second harmonic. The effectiveness of the technique has been demonstrated by measuring the Verdet constant of TGG glass and measuring the small polarization rotation angle. A sensitivity of

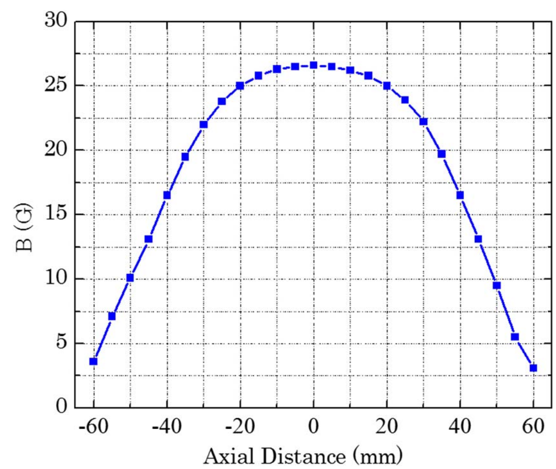

Fig. 3. Variation in the axial magnetic field of the solenoid coil of the Faraday modulator.

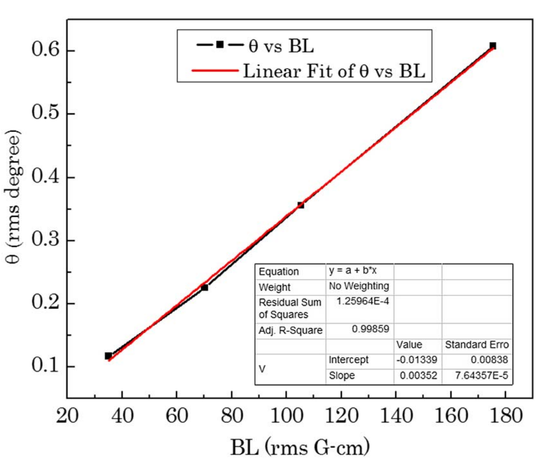

Fig. 4. Measurement of the Verdet constant of TGG magneto-optic glass using the differential measurement system.

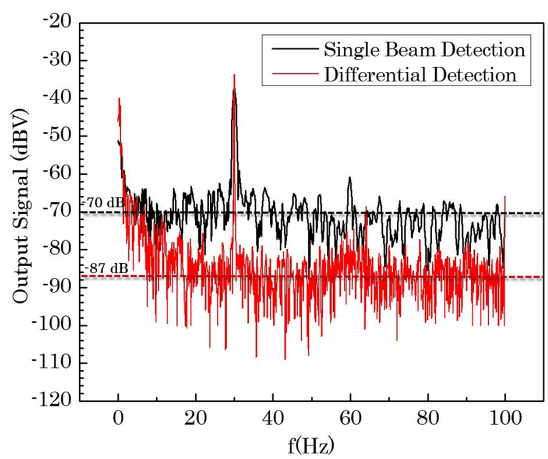

Fig. 5. Noise spectra for a small polarization rotation angle using two measurement techniques. The black and red curves correspond to the single beam detection and the optical differential detection, respectively. The low-frequency performance was considerably improved with the optical differential detection method.

the enhancement of about one order of magnitude has been achieved using differential detection based on Faraday modulation compared with the conventional single beam modulation technique.

This work was supported by the National Key R&D Program of China (No. 2017YFB0503100) and the National Science Foundation of China (NSFC) (No. 61227902).

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