

# Transmittance and phase measurement via self-calibrated balanced heterodyne detection

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It is rare for a conventional direct detection method to measure the transmittance uniformity of mirrors with rigorous standards, especially to meet the requirement of transmittance/reflectance and phase detection simultaneously. In this study, a new method of self-calibrated balanced heterodyne detection (SCBHD) is proposed. It can be self-calibrated by a two-channel structure to overcome the environmental effects in large optics scanning detection by employing highly accurate heterodyne interference. A typical transmittance measurement experiment was performed at 1053 nm wavelength via SCBHD. A standard deviation (SD) of 0.038% was achieved in the preliminary experiment. The experimental results prove to reduce the SD by approximately two orders of magnitude compared with the conventional direct detection method in the same condition. The proposed method was verified as being promising not only for its wider dynamic measurement range and its higher accuracy but also for its simultaneous transmittance and phase detection ability.

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A high-power laser facility is an important experimental platform for studying high-energy density physics, particularly inertial confinement fusion (ICF). Metrology tools such as spectrophotometers<sup>[1-6]</sup> are typically employed by both optics fabrication vendors and laboratories to ensure production control and obtain superior optical performances. Laboratories that develop those high-power laser facilities for ICF had their own metrology<sup>[7-11]</sup> for optics because of the special requirements. One of those demands is that the coating uniformity of large-scale optics should be strictly controlled<sup>[12]</sup>, such as transmittance, reflectance, or phase. The metrology of mirrors was reported by the National Ignition Facility group in 2003<sup>[8]</sup>, and full-aperture reflectance and transmittance measurements for large-scale mirrors were included. In 2007, another group associated with the experimental ICF Laser Mégajoule proposed an accurate system for optical reflectance measurement<sup>[9]</sup>, which could be realized on flat or shaped samples. Like other ICF experimental laser facilities, the “SG-II” facility is composed of thousands of optics with coatings covering an extremely wide range of transmittance or reflectance. Studies regarding the transmittance homogeneity of large-scale optics were promoted for the “SG-II” facility in China by our group in 2010, and the equipment performed well<sup>[11]</sup>. Nevertheless, one challenge that was faced is that most of the existing metrology cannot satisfy the requirements of both high accuracy during large-scale optics scanning and high precision in a wide dynamic range. The abovementioned metrology depends mainly on conventional direct detection, with a sensitivity that is limited by the shot noise, while

the precision may suffer from environmental disturbance or stray radiation scattered from the measurement system. Therefore, a new method based on other physical principles is expected to satisfy the stated requirements.

In 1988, Snyder proposed the ultrasensitive technique of heterodyne detection for measuring optical power<sup>[13]</sup> that was later verified as a powerful method for optical transmittance measurement and various other applications<sup>[14]</sup>. In principle, the heterodyne signal of interest, with a constant frequency, can be filtered and demodulated by means of an electric method. In the reported practical applications, the total photodiode current can be expressed as a function of both the local oscillator power and signal power with only one channel. The signal was demodulated in order to calculate the parameters of interest; however, noise from electric circuits and processing is apparently inevitable.

In most cases, optical power detectors such as photo-multipliers and photodiodes are used to produce an electrical current that is proportional to the incident optical power. Therefore, the dynamic range is unavoidably limited by quantum efficiency and the shot noise of the receivers. Direct detection is not good for weak-signal measurement because system errors from the receivers dominate the others and result in catastrophic errors. Hence, the dynamic range and accuracy pose the severest limitations in conventional direct detection, and an alternative method should be considered, particularly for weak signal detection.

We designed a transmittance measuring system via heterodyne detection and optical demodulation that we called self-calibrated balanced heterodyne detection

(SCBHD). Optical demodulation here was applied to upgrade the coherent heterodyne detection method, offering remarkable accuracy and extensive dynamic range in transmittance measurement. The SCBHD method involves two channels (Cos-Channel and Sin-Channel), each of which consists of two outputs with a  $90^\circ$  phase shift, as shown in Fig. 1. The principal analyses indicated that the technique offered a wider dynamic optical power measuring range and higher sensitivity compared to the conventional direct detection method. Furthermore, we found that the optical antenna properties<sup>[15]</sup> of the receivers would be dominated by system errors during the mapping of large-scale optics. The SCBHD we proposed can overcome such system errors, utilizing both orthogonal channels.

A primary system was designed for optical detection, and a coherent heterodyne detection technique was employed owing to its unprecedented dynamic range and sensitivity, particularly for weak signals<sup>[13]</sup>. The optical scheme of our setup is shown in Fig. 1. A laser beam is split into two beams by a polarization beam splitter (PBS). An appropriate frequency shift between the local and signal beams can be realized by means of a photoelastic modulator (PEM). We defined the beam with the PEM as the local oscillation beam, and the other as the signal beam. Thus, the local and signal beams, as shown in Fig. 1, can be expressed as follows by a Jones matrix:

$$\begin{aligned} E_S &= E_o(x, y, z) \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} e^{i\varphi}, \\ E_{LO} &= E_o(x, y, z) \begin{bmatrix} k_3 \\ k_4 \end{bmatrix} e^{i\phi(t)}, \end{aligned} \quad (1)$$

where  $\varphi$  is the optical phase difference between the two outgoing beams from the PBS;  $k_1$  and  $k_2$  are the parallel and perpendicular polarization components of the signal beam, respectively (while  $k_3$  and  $k_4$  are those of the local beam); and  $\phi(t)$  represents the local beam phase modulation.

The local beam passes through a quarter wave plate ( $\lambda/4$  WP) and then perpendicularly irradiates a PBS

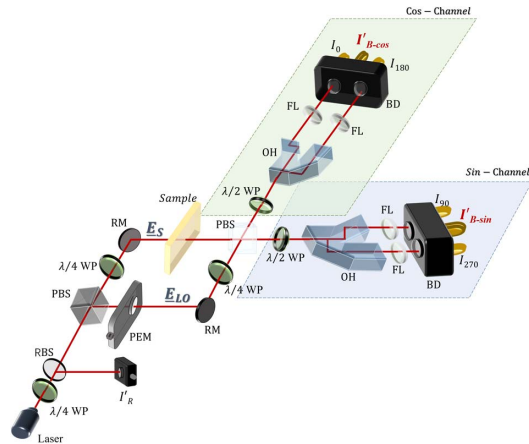


Fig. 1. Scheme for a two-channel SCBHD with a transmittance measuring sample inserted.

together with the signal beam. The situation without a sample is firstly considered. As a result, the local and signal beams interfere with each other and are split into two perpendicular polarization parts. Although two channels are illustrated in Fig. 1, in the green and blue quadrangles, respectively, one interference channel is sufficient for deducing the transmittance/reflectance of the sample. This interference light sequentially travels through a half-wave plate ( $\lambda/2$  WP) and an integrated particular optical hybrid (OH) with  $1 \times 2$  connectors; that is, the light once again effectively propagates through a PBS. As a result, the two outgoing beams differ completely from the incident ones, both in the polarization states and propagation directions.

A constant phase difference of  $180^\circ$  is produced between the two adjacent OH outputs, which is crucial for SCBHD. The focal lens (FL) aids in focusing and collimating the output beams. This kind of inherent optical demodulation can prevent complex electric processing and, as a result, the detection sensitivity may reach an inconceivable quantum limitation. Following integration over the many optical cycles corresponding to the time response of the receivers, the inference intensities of outgoing beams without a sample, as shown in Fig. 1, are expressed as follows:

$$\begin{aligned} I_0 &= \frac{1}{2} (k_2^2 + k_3^2) |E_o|^2 + k_2 k_3 |E_o|^2 \cos \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \\ I_{180} &= \frac{1}{2} (k_2^2 + k_3^2) |E_o|^2 - k_2 k_3 |E_o|^2 \cos \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \\ I_{90} &= \frac{1}{2} (k_1^2 + k_4^2) |E_o|^2 - k_1 k_4 |E_o|^2 \sin \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \\ I_{270} &= \frac{1}{2} (k_1^2 + k_4^2) |E_o|^2 + k_1 k_4 |E_o|^2 \sin \left( \phi(t) - \frac{\pi}{4} - \varphi \right). \end{aligned} \quad (2)$$

The outputs of balanced detectors (BDs) are the heterodyne results of the two inputs with a constant phase difference. The BDs' heterodyne outputs are formulated as follows:

$$\begin{aligned} I_{B-\cos} &= I_0 - I_{180} = 2k_2 k_3 |E_o|^2 \cos \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \\ I_{B-\sin} &= I_{270} - I_{90} = 2k_1 k_4 |E_o|^2 \sin \left( \phi(t) - \frac{\pi}{4} - \varphi \right). \end{aligned} \quad (3)$$

Taking transmittance measuring as an example, which is the most typical coating homogeneity measurement, the sample is inserted into the signal beam, as shown in Fig. 1. As the transmittance  $T$  of the sample is introduced, the signal beam with the sample inserted can be expressed as follows:

$$E'_S = \sqrt{T} E_o(x, y, z) \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} e^{i\varphi'}. \quad (4)$$

The initial optical phase difference  $\varphi'$  is constant and can be ignored in this case. The amplitude outputs from

the BDs, which are valuable for the transmittance measurement, can be rewritten as

$$\begin{aligned}\langle I_{B-\cos} \rangle &= \langle I_0 - I_{180} \rangle = 2k_2k_3|E_o|^2 = 2k_2k_3 \cdot \alpha I_R^2, \\ \langle I_{B-\sin} \rangle &= \langle I_{270} - I_{90} \rangle = 2k_1k_4|E_o|^2 = 2k_1k_4 \cdot \alpha I_R^2, \\ \langle I'_{B-\cos} \rangle &= 2k_2k_3\sqrt{T}|E'_o|^2 = 2k_2k_3\sqrt{T} \cdot \alpha I_R^2, \\ \langle I'_{B-\sin} \rangle &= 2k_1k_4\sqrt{T}|E'_o|^2 = 2k_1k_4\sqrt{T} \cdot \alpha I_R^2,\end{aligned}\quad (5)$$

where  $I'_{B-\cos}$  and  $I'_{B-\sin}$  are the outputs with the sample inserted and  $I_{B-\cos}$  and  $I_{B-\sin}$  are those without, and  $I_R$  and  $I'_R$  refer to the reference beam with and without the sample inserted, respectively. A reference beam reflected by a coated beam splitter (CBS) is also necessary for calibration, as illustrated in Fig. 1. The transmittance can be calculated simply by using the current amplitudes if source fluctuation is neglected, as follows:

$$T = \frac{\langle I'_{B-\cos} \rangle^2}{\langle I_{B-\cos} \rangle^2} = \frac{\langle I'_{B-\sin} \rangle^2}{\langle I_{B-\sin} \rangle^2} = \frac{\langle I'_{B-\cos} \rangle \cdot \langle I'_{B-\sin} \rangle}{\langle I_{B-\cos} \rangle \cdot \langle I_{B-\sin} \rangle}.\quad (6)$$

While the laser source fluctuation is considered and only one channel is applied, or two channels are applied in order to avoid system fluctuation, the calibrated transmittance  $T_C$  can be calculated as follows, considering source power variation:

$$\begin{aligned}T_C &= \frac{\langle I'_{B-\cos} \rangle^2 \cdot I_R^2}{\langle I_{B-\cos} \rangle^2 \cdot I_R^2} = \frac{\langle I'_{B-\sin} \rangle^2 \cdot I_R^2}{\langle I_{B-\sin} \rangle^2 \cdot I_R^2} \\ &= \frac{\langle I'_{B-\cos} \rangle \cdot \langle I'_{B-\sin} \rangle \cdot I_R^2}{\langle I_{B-\cos} \rangle \cdot \langle I_{B-\sin} \rangle \cdot I_R^2}.\end{aligned}\quad (7)$$

The transmittance can be calibrated with either one channel with a reference beam, or two channels. Obviously, the setup including two channels is preferable, because only the source disturbance is considered in the one-channel setup, and the system error can be removed by conjugate beams and two identical BDs. Further self-calibration between two channels is stated with the misalignment error coefficient in the latter paragraph.

It is also appropriate for the reflectance measurement to mirror at 45°, which can be achieved by simply changing the mirror in the signal beam to the sample. The reflectance of the sample can be calculated using Eq. (7), considering the reflected mirror (RM) in the local beam as an ideal total reflected mirror. Of course, suitable collimation and data calibration are necessary to provide accurate results. However, in order to obtain high accuracy, it is preferable to calibrate the RM reflectance. The calibrated reflectance of a sample  $R_C$  is

$$R_C = \frac{\langle I'_{B-\cos} \rangle \cdot \langle I'_{B-\sin} \rangle \cdot I_R^2}{\langle I_{B-\cos} \rangle \cdot \langle I_{B-\sin} \rangle \cdot I_R^2} \cdot R_L,\quad (8)$$

where  $R_L$  refers to the calibrated reflectance of the RM as a standard mirror in the load beam, which mainly determines the accuracy of the results.

Phase disturbance, which is often neglected, is an additional key parameter affecting the damage threshold of large optics in high-power laser facilities<sup>[15]</sup>. The existing transmittance detection utilizes direct power detection in most cases; thus, phase detection cannot be realized. The setup we constructed can measure the transmittance and phase simultaneously. Optics such as windows, mirrors, or other types of glass with a small phase disturbance can be expressed by means of a Jones matrix, as follows:

$$S = \sqrt{T} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta} \end{bmatrix}.\quad (9)$$

Eventually, the outputs of the BDs are

$$\begin{aligned}I''_{B-\cos} &= I''_0 - I''_{180} \\ &= 2k_2k_3\sqrt{T}|E_o|^2 \cos\left(\phi(t) - \frac{\pi}{4} - \varphi - \Delta\right), I''_{B-\sin} \\ &= I''_{270} - I''_{90} = 2k_1k_4\sqrt{T}|E_o|^2 \sin\left(\phi(t) - \frac{\pi}{4} - \varphi\right).\end{aligned}\quad (10)$$

Therefore, the phase of sample  $\Delta$  can be determined according to the phase deviation of the two channels. The phase detection results can be optimized by curve fitting over several optical cycles. The transmittance calculation was previously introduced. The conclusion can be drawn that our setup can realize transmittance/reflectance and phase detection simultaneously.

The SCBHD method is logically superior to direct detection because advanced optical demodulation is employed instead of the conventional electrical technique, and sensitivity can be reduced to the quantum level influenced by optical fluctuations in the signal field<sup>[13]</sup>. In this case, the noise equivalent power is expressed as  $NEP = \frac{hw\Delta f}{\eta}$ , where  $\Delta f$  is the detection bandwidth and  $\eta$  is the detector quantum efficiency. Accordingly, the dynamic range of optical heterodyne detection is expressed as  $R_h = P_{\text{sat}}/NEP$ , where  $P_{\text{sat}}$  is the maximum saturated detector power. To the best of our knowledge, applications often involve milliwatt-level power detection, and assuming a detector quantum efficiency of  $\eta \approx 1$  the detection bandwidth range is approximately several hertz to several megahertz. The sensitivity is approximate to the NEP when the signal beam is far weaker than the local beam. Thus, the theoretical sensitivity is approximately  $10^{-15}$ – $10^{-9}$  W and the estimated SCBHD dynamic range is approximately  $R_h = 10^9$ – $10^{15}$ , which is far greater than that of direct detectors.

The abovementioned optical balanced coherent detection scheme allows for accurate measurements with high sensitivity and a dynamic range. However, raster scanning of large-scale optics requires a high-precision setup between each scanning point. Therefore, the antenna properties of the receivers become the main cause of the system errors.

According to Eq. (3),  $I_{B-\cos}$  and  $I_{B-\sin}$  can be expressed as

$$\begin{aligned} I_{B-\cos} &= K_1 \sqrt{\alpha_1 \alpha_2} \cos \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \\ I_{B-\sin} &= K_2 \sqrt{\alpha_3 \alpha_4} \sin \left( \phi(t) - \frac{\pi}{4} - \varphi \right), \end{aligned} \quad (11)$$

where  $\alpha_1, \alpha_2, \alpha_3$ , and  $\alpha_4$  are the coefficients proportional to the effective areas of the four receivers in two channels,  $K_1 = 2k_2 k_3 |E_o|^2$  and  $K_2 = 2k_1 k_4 |E_o|^2$ .

As the two interference beams travel through the nearly symmetrical light paths, the main difference certainly lies in the effective receiver apertures of the two OHs. This is because raster scanning can result in beam shifting and consequently misalignment, which can be ignored only when the receiver aperture is significantly larger than the beam aperture and beam shifting does not cause misalignment; that is,  $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$ . However, in the practical experimental setup, the negative effect certainly exists and can cause disastrous effects.

While a single point detection is concerned, the ratio of the effective coefficients of two interference beams is constant; thus, the representative parameter of the misalignment error is expressed as

$$\beta_0 = |I_{B-\cos}/I_{B-\sin}| = \frac{k_2 k_3 \cdot \sqrt{\alpha_1 \alpha_2}}{k_1 k_4 \cdot \sqrt{\alpha_3 \alpha_4}}. \quad (12)$$

In raster mapping of a large-scale mirror, there are system errors between different test points, according to the previous interpretation. Taking the transmittance measurement for example, we denote the transmittance of the first point as  $T_1$  and consider the error parameter  $\beta_0$  at point 1 as the initial system error. Moreover,  $\beta_{n-1}$  is the system error parameter at point  $n$ . The ratio of  $\beta_1$  to  $\beta_0$  indicates the difference in system error from points 1 to 2, and if  $n = 1, 2, \dots$ , this misalignment error ratio is expressed as

$$\Delta\beta_n = \beta_n / \beta_{n-1}. \quad (13)$$

$\Delta\beta_n$  is only related to the variation of  $\alpha_n$  from each point, or the outputs of the two channels in other words, according to Eq. (13). As a result, the measured transmittance  $T_{n+1}$ , which is shown in Fig. 2, should be calibrated according to  $\Delta\beta_n$ , as follows:

$$T'_{n+1} = T_{n+1} \Delta\beta_n. \quad (14)$$

Therefore, it is obvious that the unavoidable system error induced by effective receiver aperture variation during raster scanning detection can be decreased by such calibrations.

Some experiments were done to compare with direct detection, and SCBHD without calibration which we named as optical balanced heterodyne detection (OBHD). The SCBHD was shown to be better than both. We furthermore proposed an improved setup to detect phase

variation, by replacing the ordinary beam splitter (BS) with a polarization one, which is essential for large optics in high-power lasers and is related to the wavefront, damage threshold, and other parameters<sup>[16–18]</sup>.

An experiment was performed to determine the coherent detection of transmittance. A semiconductor laser with a wavelength of 1053 nm was used as the coherent source for the measurement. The PEM was manufactured by Hinds Instruments. In optical balanced detection, the frequency is always designed to be lower than the photodiode bandwidth, which is 100 kHz in our experiment. Coated silica glass is designed and fabricated to serve as the OHs, which are required to exhibit identical optical performances. The two identical BDs (PDB450 C) produced by Thorlabs can receive light effectively in the wavelength range of 800 to 1700 nm.

In order to demonstrate the viability of the proposed method, a  $\Phi 50$  mm sample made of K9 glass with coating was measured, and its transmittance spectrum was calibrated in advance using a commercial photometer, as illustrated in Fig. 3. The exact transmittance at the 1053 nm wavelength is 95.13%.

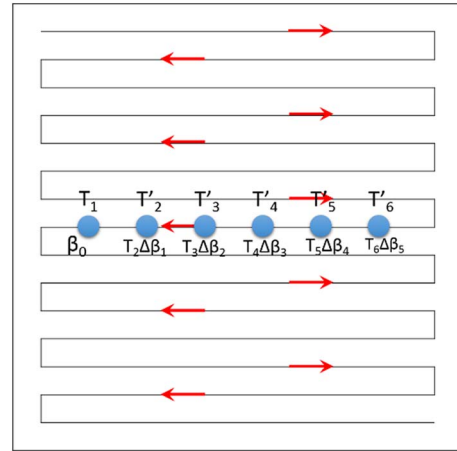


Fig. 2. Schematic of the transmittance measurement with different system errors.

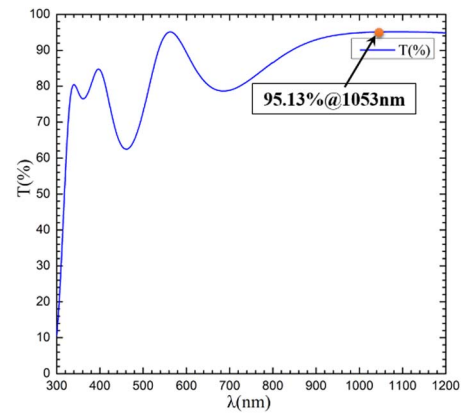


Fig. 3. Transmittance spectrum of the K9 glass sample measured using a commercial photometer.



We detected at one point with the help of the motor driven translation to study the repeatability of the method. When the sample was inserted into the signal beam, the coherent detection was performed on a movable sample, and transmittance was tested at one point of the sample. The sample was moved by one step to the left and then one step back to the right, before each detection was performed. Thus, we considered that the detection was performed on the fixed point, and the results should be constant in theory. Based on Eq. (13), the self-calibrated misalignment error coefficients  $\Delta\beta_n$  are listed in Fig. 4. According to the calculated  $\Delta\beta_n$ , the measured and calibrated heterodyne signals are shown in Fig. 5. Clearly, the proposed SCBHD calibration method can effectively reduce system errors that are mainly due to misalignment.

As previously mentioned, the proposed SCBHD calibration method is particularly beneficial for the movement measurement of large-scale optics, with the help of the misalignment error coefficient  $\Delta\beta_n$ . In order to verify

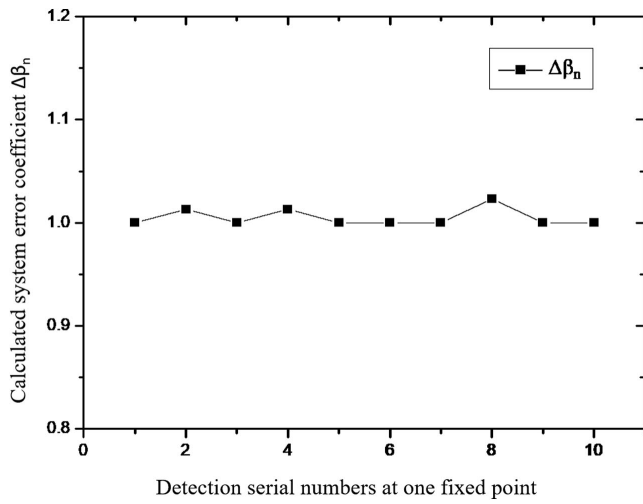


Fig. 4. Calculated misalignment error coefficient  $\Delta\beta_n$  for the detected heterodyne signals.

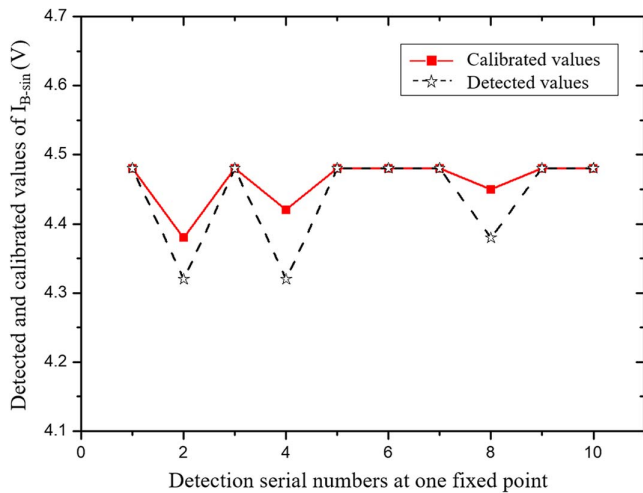


Fig. 5. Detected heterodyne signal  $I_{B-\sin}$  and its calibrated values.

the validity of the method, a 2D scanning system was controlled using a computer, and the scan increment was in the order of 50 mm. Ten demo points on the sample were tested via direct detection, one-channel OBHD, and two-channel SCBHD, respectively, as shown in Fig. 6. The SDs for direct detection, OBHD, and SCBHD are 1.25%, 0.052%, and 0.038%, respectively.

Experiments of phase measurement were performed to validate the proposed technique. A standard  $\lambda/4$  WP was chosen as the testing sample assuming it as an ideal one, and its phase was measured to be  $91.001^\circ$  by SCBHD. The feasibility of the proposed method to measure phase retardation was preliminarily proved according to the results. But more study should be done in the future to analyze and calibrate the system errors to improve its accuracy and repeatability. There are many influencing factors, such as system misalignment, source disturbance, vibration, and environmental temperature.

In summary, an SCBHD setup was constructed for simultaneous measurements of the laser transmittance and its phase disturbance by applying the balanced heterodyne theory. Such a technique fulfills the measurement requirements of the extremely wide dynamic range in examining the large optical elements of high-power lasers for the ICF research. Comparison experiments indicate that the proposed method achieved precise measurements with an SD of 0.038%, which surpassed that of direct detection in nearly the same detection situation by approximately two orders of magnitude. The two-channel structure of SCBHD can be self-calibrated to reduce the system error caused by one channel, and to reduce misalignment errors that deeply harm the raster scanning detection with the help of misalignment error coefficient  $\Delta\beta_n$ . Furthermore, this method is demonstrated to be superior to the existing metrology in terms of simultaneous phase detection, which is another important parameter for high-power laser facilities. Analyses of the theoretical sensitivity and the estimated dynamic range indicate that SCBHD is far greater

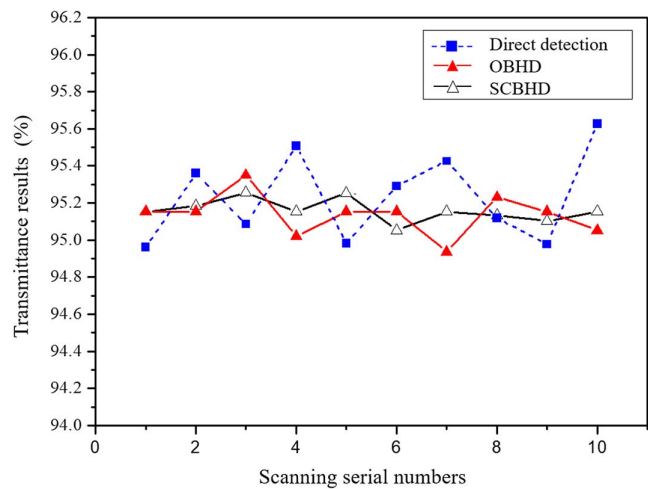


Fig. 6. Transmittance measured via OBHD, SCBHD, and direct detection.

than that of direct detectors. The demonstrative experiments of phase measurement were also performed, and the validity of phase measurement was preliminarily proved. More applications and system errors are still being studied to improve its performance, especially in phase detection, but the proposed method was verified as being prospective to meet the rigorous optics measurement demands of large optics in the ICF study, and the simultaneous measurement of two parameters can tremendously cut down testing time.

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### References

1. D. Mielenz, K. L. Eckerle, R. P. Madden, and J. Reader, *Appl. Opt.* **12**, 1630 (1973).
2. D. Enard and H. Visser, *Appl. Opt.* **21**, 4459 (1982).
3. A. Bittar and J. D. Hamlin, *Appl. Opt.* **23**, 4054 (1984).
4. D. D. Sell, *Appl. Opt.* **9**, 1926 (1970).
5. D. Enard and H. Visser, *Appl. Opt.* **21**, 4459 (1982).
6. E. I. Moses, J. H. Campbell, C. J. Stolz, and C. R. Wuest, *Proc. SPIE* **5001**, 1 (2003).
7. M. L. André, *Fusion Eng. Des.* **44**, 43 (1999).
8. J. Stolz, M. Runkel, M. S. McBurney, R. E. Cheek, and J. A. Menapace, *Proc. SPIE* **5341**, 114 (2004).
9. S. Petitrenaud, P. Voarino, H. Piombini, F. Sabary, and D. Marteau, *Appl. Opt.* **46**, 8609 (2007).
10. H. Wang, Z. Da, and L. Liu, *Appl. Opt.* **51**, 2395 (2012).
11. J. Miao, P. Yang, B. Zhu, and J. Zhu, *Proc. SPIE* **7656**, 76564K (2010).
12. H. Qi, M. Zhu, M. Fang, S. Shao, C. Wei, K. Yi, and J. Shao, *High Power Laser Sci. Eng.* **1**, 36 (2013).
13. J. J. Snyder, *Appl. Opt.* **27**, 4465 (1988).
14. A. L. Migdall, B. Roop, Y. C. Zheng, J. E. Hardis, and G. J. Xia, *Appl. Opt.* **29**, 5136 (1990).
15. A. E. Siegman, *Appl. Opt.* **5**, 1588 (1966).
16. X. Chen, X. Li, Z. Chen, J. Pu, G. Zhang, and J. Zhu, *High Power Laser Sci. Eng.* **1**, 132 (2013).
17. B. Wang, J. Zhang, S. Shi, K. You, and J. Zhu, *High Power Laser Sci. Eng.* **4**, e9 (2016).
18. X. Zhang, W. Zhou, W. Dai, D. Hu, X. Deng, W. Huang, L. Zhou, Q. Yuan, X. Huang, D. Wang, and Y. Yang, *High Power Laser Sci. Eng.* **4**, e6 (2016).