

Effects of intermittency and stratification on the evaluation of optical propagation

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To evaluate the effects of intermittency and stratification on propagation of laser beams, we analyze experimental radiosonde profiles using complete ensemble empirical mode decomposition. First, we extract intermittency and stratification at different scales from the original C_n^2 profiles. Second, we establish an intrinsic optical turbulence model (IOTM), which includes fine structures. Third, we calculate several turbulence moments using IOTM. The results quantitatively evaluate the effects of variations at different scales on typical parameters relevant to optical turbulence effects.

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The characteristics of atmospheric media influence the propagation of laser beams, leading to a variety of optical turbulence effects^[1-3]. The strength of optical turbulence is measured by the refractive index structure parameter C_n^2 , and the optical turbulence effects are mainly governed by the distribution of C_n^2 on the propagation path. Observations have revealed that C_n^2 varies with height, characterized by bursty extremum and distinct layers^[4-9], which, experimentally, are symbols of intermittency and stratification structures, respectively. As the shape of C_n^2 profiles bring influence to optical turbulence effects, it is suggested that an appropriate optical turbulence model would be one where stratification structures are superposed on a background optical turbulence. However, hindered by scarcity of theoretical models and valid statistical methods, the current assessment of optical turbulence effects usually rely on smoothed models generally ignoring the fine structures^[9-12].

In this Letter, we introduce a novel method named complete ensemble empirical mode decomposition (CEEMD) to process high-resolution optical turbulence profiles. Then, we establish a novel optical turbulence model named the intrinsic optical turbulence model (IOTM), and further evaluate the effects of intermittency and stratification structures on optical turbulence effects.

Among all of the techniques used to measure the C_n^2 profile (DIMM, GSM, SCIDAR, MASS and SLODAR)^[13], a radiosonde balloon equipped with microthermal and meteorological sensors has become the standard and most delicate facility. The microthermals can measure the C_n^2 profiles with an exceptionally high vertical resolution, which give a precise idea of the shape of the turbulent

layers. The microthermal sensors used in our experiments were developed at the Key Laboratory of Atmospheric Composition and Optical Radiation, Chinese Academy of Sciences. During a flight, the microthermal sensors probe and calculate the temperature structure constant C_T^2 ($\text{K}^2 \text{m}^{-2/3}$), using

$$C_T^2 = \left\langle \left[T(x) - T(x+r) \right]^2 \right\rangle \cdot r^{-2/3}, \quad l_0 \ll r \ll L_0, \quad (1)$$

where the triangle brackets denote an ensemble average, r is the separation between two temperature probes in the position of x and $x+r$, l_0 is the inner scale, and L_0 is the outer scale that define the lower and upper limit of the inertial range. The noise level of the probes in microthermal sensors corresponds to a temperature difference of 0.002 K, implying a minimum detectable C_T^2 of $4.0 \times 10^{-6} \text{K}^2 \text{m}^{-2/3}$. The meteorological radiosonde measures the ambient temperature, pressure, and humidity, and GPS is used to obtain position, wind velocity, and wind direction. The detectable precision of the temperature is 0.2 K, and the precision of the pressure is 1.5 hPa. Figure 1 gives the simultaneously measured temperature, pressure, and C_T^2 at noon on 4 November 2015.

As a function of height h , optical turbulence strength C_n^2 ($\text{m}^{-2/3}$) is an expression of absolute temperature T (K), pressure P (hPa), and temperature structure constant C_T^2 ^[1]:

$$C_n^2(h) = \left[79 \times 10^{-6} \frac{P(h)}{T^2(h)} \right]^2 C_T^2(h). \quad (2)$$

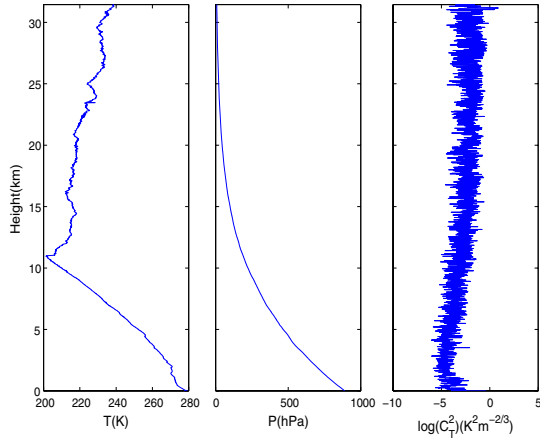


Fig. 1. Profile of (a) temperature, (b) pressure, and (c) C_n^2 .

During November 2015, we launched three balloons on each sunny day within 1–2 h of local time 8, 14, and 20 o'clock in the Northwest Plateau of China. Figure 2 shows the C_n^2 profile at noon on 4 November 2015. One can see that C_n^2 fluctuates intermittently around the mean, which is averaged over a scale of 1000 m. The original data represented by gray dots illustrate the frequent fluctuations due to intermittency, and the variation of the red mean line shows the stratification structures.

The calculation of C_n^2 , based on measured C_T^2 , is subjected to uncertainties in pressure, temperature, and C_T^2 . Assuming that these errors are uncorrelated with each other, the uncertainty analysis of Eq. (2) results in the estimate of the error in C_n^2 ,

$$\left(\frac{\sigma}{C_n^2}\right)^2 = 4\left(\frac{\sigma_P}{P}\right)^2 + 16\left(\frac{\sigma_T}{T}\right)^2 + \left(\frac{\sigma_{C_T^2}}{C_T^2}\right)^2, \quad (3)$$

where σ represents the error of C_n^2 , σ_P , σ_T and $\sigma_{C_T^2}$ are the uncertainties in pressure, temperature, and C_T^2 , respectively. As shown in Fig. 1(c), $\log(C_T^2)$ varies around -5 at low altitudes and increases with height, staying around -2 in the upper air.

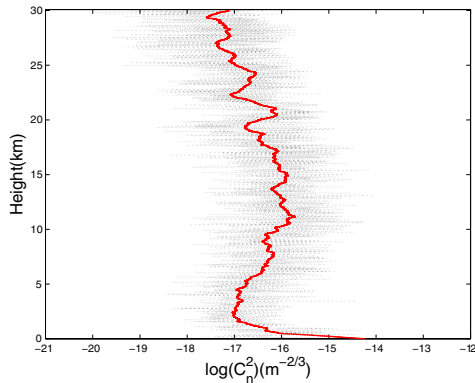


Fig. 2. Refractive index structure constant C_n^2 profile recorded at noon on 4 November 2015.

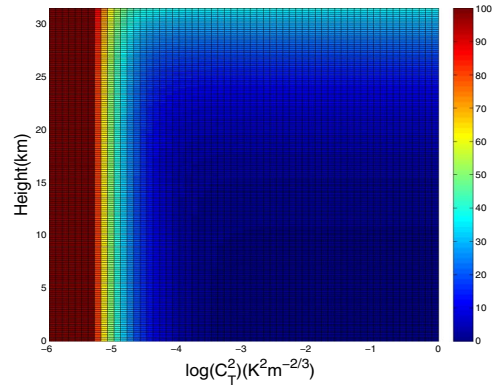


Fig. 3. Relative error ($\sigma/C_n^2 \times 100\%$) estimates in C_n^2 as a function of height and $\log(C_T^2)$.

Figure 3 shows the relative error σ/C_n^2 in C_n^2 of the profile in Fig. 2, plotted against the height and $\log(C_T^2)$. The value of σ/C_n^2 decreases with the increase of C_T^2 , and increases with the height as the temperature and pressure fall off. The uncertainty in C_n^2 is about 40% at low altitudes, about 10% at modest altitudes, and increases to about 30% above 25 km.

Analyzing the nonlinear and nonstationary series like C_n^2 profiles, traditional data-analysis methods all suffer one or more flaws, such as predetermined assumptions and the questionable approximation of ensemble mean using temporal or spatial mean. In 1998, Huang *et al.*^[14] proposed empirical mode decomposition (EMD) to eschew the defects of traditional approaches. Latterly, Wu and Huang^[15] developed a noise-assisted technique called ensemble EMD (EEMD) to solve the mode mixing problem of EMD, but it created new ones. Torres *et al.*^[16] recently upgraded EEMD to CEEMD, which includes a negligible error and less computational cost. EMD and its upgraded versions have become an established tool for time-frequency analysis of nonlinear and nonstationary signals in recent years^[17,18]. EMD decomposes a signal into several oscillating intrinsic mode functions (IMFs) by a process called the sifting algorithm. Based on EEMD, CEEMD adds white noise at each stage of the decomposition and computes a unique residue as each mode. At the end, a given signal x can be represented as a sum of IMFs and a residue R ,

$$x = \sum_{i=1}^K c_i + R, \quad (4)$$

where K is the total number of IMFs, c_i the i th IMF, R is also named as a trend term. As $\log(C_n^2)$ rather than C_n^2 is generally used in applications, optical or C_n^2 profiles refer to the spatial series of $\log(C_n^2)$ specifically hereinafter.

Figure 4 shows the CEEMD decomposition result of the C_n^2 profile in Fig. 2, which is decomposed into nine IMFs and a residue. The low-order IMFs contain the highest and strongest fluctuations, while the subsequent ones contain smaller and slower fluctuations.

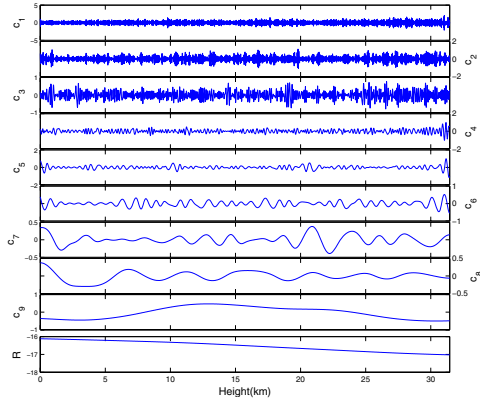


Fig. 4. IMFs for a C_n^2 profile.

Indeed, we have examined totally 25 C_n^2 flights under various meteorological conditions, which are valid after washing out the bad data, and have demonstrated the suitability of CEEMD. For all of the decompositions, 4 profiles produce 8 IMFs, 15 profiles produce 9 IMFs, and the rest produce 10 IMFs. We have found the physical meaning of every IMF, which is consistent with other studies^[19]. Though the number and shape of IMFs are different between different profiles, we find that the fourth to 8 IMFs in all of the profiles are associated with temperature gradients and wind shears. Our experiments indicate that the results of CEEMD are insensitive to the amplitude of the added noise. Besides, a large realization is suggested to get stable numbers of the modes. We set the level of noise as 0.2 multiples to standard deviation of the input signal, the number of realizations as 2000, and the maximum sifting iterations as 2000.

Wu and Huang established a statistical significance test method for IMFs against white noise^[20]. Assuming the first IMF as a referenced noise, the averaged period of each IMF and its energy density are plotted in Fig. 5. Only those lying above the 80% lines contain physical meaning. It is clear that the second and third IMFs likely contain little useful statistically significant information. Most of the fourth and all of the rest of the IMFs locate above the 80% confidence, indicating that they contain significantly quasi-cyclical variations with height.

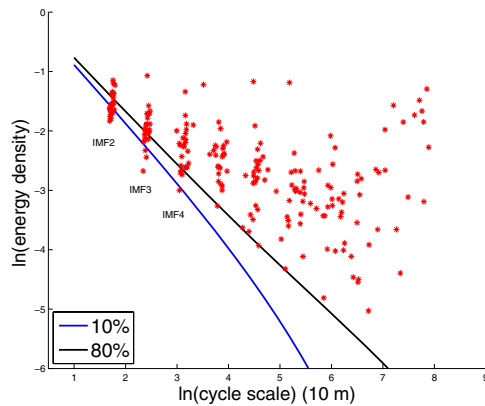


Fig. 5. (Color online) Significant test results for the IMFs of 25 C_n^2 profiles.

The contribution of IMFs at different scales on the whole profile can be evaluated quantitatively by the variance contribution rate (VCR). The VCR of the i th IMF component VCR_i can be calculated via the formula

$$VCR_i = \frac{\text{var}(c_i)}{\sum_{j=1}^K \text{var}(c_j) + \text{var}(R)} \times 100\%. \quad (5)$$

Table 1 lists the mean and standard deviation of quasi-cyclical scales and the VCR of IMFs. We can see that the first three IMFs maintain frequent variations with scales smaller than 113 m, contributing about 50% to whole profiles. The scales of IMF4 – IMF8 range from 232 to 4244 m with little VCR. IMF9 and IMF10 are large-scale variations whose VCR is 31%.

Thus, we get the scales and statistical significance of each IMF. Judging from the statistical significance test and quasi-cyclical scale, we believe that the first three IMFs are intermittent oscillations, and the quasi-cyclical IMFs maintain variations due to stratification structures.

Based on CEEMD, we obtain an IOTM, which includes stratification structures by superimposing specific scale of IMFs. Note the $\log(C_n^2)$ series as x in Eq. (4), P_k as the k -order profile at the chosen scale of the IOTM,

$$P_k = c_k + c_{k+1} + \dots + R. \quad (6)$$

Obviously, P_1 is the original profile, while P_k holds intermittency in the k th IMF and subsequent ones. Figure 6(a) gives four intrinsic optical turbulence profiles of the original profile in Fig. 2. We can add stratification structures into the model referring to the scales in Table 1. Figure 6(b) plots a comparison between P_7 , P_9 , and running mean profiles. Obviously, the variation of P_7 is similar to that of the mean profile over 1 km, while P_7 corresponds to a mean profile over 10 km.

Table 1. Mean and Standard Deviation of Cyclical Scale and VCR for Each IMF

Component	Scale (m)		VCR (%)	
	Mean	S. D. ^a	Mean	S. D.
IMF1	28	0.4	31	9.6
IMF2	57	1.7	12	3.1
IMF3	113	3.6	8	1.9
IMF4	232	15.0	6	1.7
IMF5	457	43.7	5	1.3
IMF6	949	80.7	4	1.4
IMF7	1979	280.4	3	1.1
IMF8	4244	927.2	2	1.2
IMF9	11768	6791.2	31	15.4
IMF10	19711	4132.3	—	—

^aS. D., standard deviation.

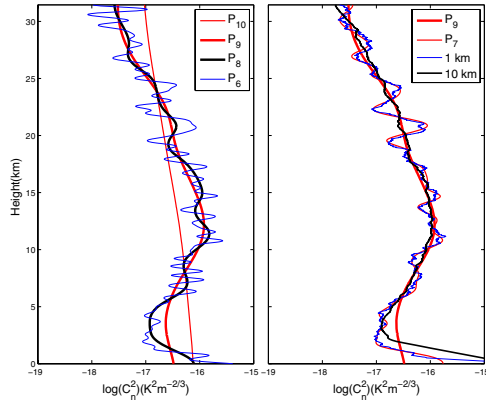


Fig. 6. (Color online) (a) Intrinsic optical model. (b) Comparison between intrinsic and running mean profiles.

To describe wave propagation in atmosphere, one has to use a comprehensive description of the media that is as realistic as possible. Fine structures are a natural feature of the C_n^2 signal, which lead to a discrepancy between the averaged model and measured signal^[2]. In adaptive optics and site-testing engineering, the coherent length and isoplanatic angle are pivotal parameters to the design and evaluation of systems. For plane waves translating vertically from height $h = 0$ to $h = L$, coherence length r_0 and isoplanatic angle θ_0 are expressed as^[1]

$$r_0 = \left(0.423 k^2 \int_0^L C_n^2(h) dh \right)^{-3/5}, \quad (7)$$

$$\theta_0 = \left(2.914 k^2 \int_0^L C_n^2(h) h^{5/3} dh \right)^{-3/5}. \quad (8)$$

Here, wavenumber $k = 2\pi/\lambda$, and wavelength $\lambda = 0.5 \times 10^{-6}$ m.

Figure 7 shows the coherence length and isoplanatic angle of the C_n^2 profile in Fig. 2, calculating from the IOTM. The value of r_0 raises from 2.30 at P_1 to 5.48 at P_4 , finally reaching 7.72 at P_{10} . The value of θ_0 raises from 0.74 at P_1 to 1.31 at P_4 , finally reaching 1.97 at P_{10} . Note that r_0 and θ_0 increase drastically with the deletion of IMF9 to the model, emphasizing the importance of the commonly detected bulge at the tropopause for coherence length and isoplanatic angle. It is interesting that though coherence length is sensitive to low height turbulence, and the isoplanatic angle is sensitive to upper air turbulence, they both fall off when stratification structures are added into the C_n^2 model.

The calculations of optical turbulence effects are always related to the integrals of C_n^2 weighed by height, which are called turbulence moments^[11]. Here, we consider the turbulence moment I_1 , I_2 , and I_3 , corresponding, respectively, to coherence length, isoplanatic angle, and scintillation^[1]:

$$I_1 = \int_0^L C_n^2(h) dh, \quad (9)$$

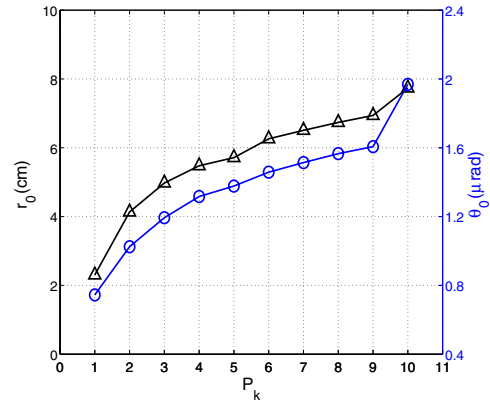


Fig. 7. (Color online) Coherence length (black triangle) and isoplanatic angle (blue circle) calculated using the IOTM.

$$I_2 = \int_0^L C_n^2(h) h^{5/3} dh, \quad (10)$$

$$I_3 = \int_0^L C_n^2(h) (L-h)^{5/6} dh. \quad (11)$$

The contributions of different intrinsic optical turbulence profiles to I_1 , I_2 , and I_3 can be quantified by the ratio

$$m_1(k) = \int_0^L 10^{P_k(h)} dh / I_1 \times 100\%, \quad (12)$$

$$m_2(k) = \int_0^L 10^{P_k(h)} h^{5/3} dh / I_2 \times 100\%, \quad (13)$$

$$m_3(k) = \int_0^L 10^{P_k(h)} (L-h)^{5/6} dh / I_3 \times 100\%. \quad (14)$$

The blue dots in Fig. 8 show a scatter plot of m_1 , m_2 , and m_3 versus the order of intrinsic optical turbulence profiles. For convenience, the 10th IMF and the residue are amalgamated into the ninth one and can be regarded as the large-scale background profile. Generally, at a given order, m_1 , m_2 , and m_3 of the 25 different profiles are highly variable due to the diverse distribution of different profiles. However, in most of the cases, they vary in the same decreasing trend. Judging from the mean level that is represented as the red line, m_1 drops down to 25.6% at P_4 , highlighting the impact of intermittent parts on propagation. m_1 decreases from 21.1% at P_5 to 15.9% at P_8 , showing the contribution of stratification structures. P_9 , regarded as the background profile, provides a nearly 13.4% contribution to I_1 . I_2 and I_3 share a similar trend with a minor difference in value.

In conclusion, we affirm the feasibility of CEEMD for analyzing optical turbulence profiles. The established IOTM successfully adds fine structures into the optical turbulence model. The calculations of turbulence moments using the IOTM show the effects of intermittency and stratification. Future investigations are needed to

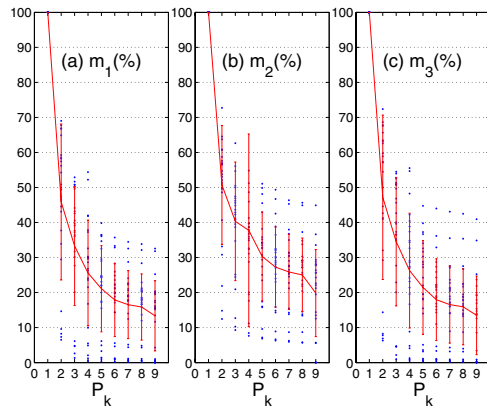


Fig. 8. Ratio of turbulence moments calculated using the IOTM to those using the original profile. (a) m_1 corresponding to coherence length. (b) m_2 corresponding to isoplanatic angle. (c) m_3 corresponding to scintillation.

verify the results with other measuring methods such as the differential image motion monitor.

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