# Heterodyne direct-detection OOFDM system with low sampling rate

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Received June 1, 2010

A method of multi-channel receiving for high bit rate heterodyne direct-detection optical orthogonal frequency-division-multiplexing (OOFDM) system is proposed to reduce the sampling rate demand of analog-to-digital converter (ADC). The sampling rate of ADCs can be reduced to $1/N$ that of the original signal bandwidth in an $N$-channel receiving system. Aided by a succeeding digital signal processing (DSP) at the receiver, aliasing free signal can be recovered. A back-to-back experimental result is given for a 4-channel system, based on which, a down conversion process for heterodyne can be reduced. The signal rebuilding algorithm is given and analyzed in its complexity and noise tolerance.

*OCIS codes:* 060.0060, 060.4080, 060.4230.

*doi: 10.3788/COL20100809.0819.*

Optical orthogonal frequency-division-multiplexing (OOFDM) has been an excellent candidate for broadband optical transmission because of its superior performance attributed to optical signal-to-noise ratio (OSNR) requirement, dispersions tolerance, and spectral efficiency[1]. Different from OFDM in radio communication, OOFDM can be applied in a much wider band. With the bit rate rising to 100 Gb/s or beyond and before the maturity of all OOFDM[2], the sampling rate of electrical parts is still the major limit in the whole OOFDM system, in which the digital-to-analog (DA) and analog-to-digital (AD) devices are the main barriers to conquer. Some methods to reduce DA sampling rate limit are reported[3], but few are brought up regarding AD solutions[4]. To solve this problem, we brought up a method by using multiple low bandwidth receivers and pre-distorted signals in Ref. [5] and proved it experimentally[6].

In this letter, specific to the direct, discrete, and linear expression of information in frequency domain of OFDM signal, we present and demonstrate a method that splits and superposes the broadband discrete spectra into a narrow band. It cooperatively utilizes multiple channels of AD converter (ADC) and fast Fourier transform (FFT) units with low sampling rate and small size. The signal can be recovered in the succeeding digital signal processing (DSP) with a linear operation. Moreover, using this method, the conventional down conversion process based on hardware for heterodyne can be omitted because the band-pass sampling process realizes a down conversion.

As shown in Fig. 1, in an $N$-channel receiving system, denote the subcarriers of the OFDM symbol as $S(K)$ ($1 \leq K \leq n_c$ with a length of $n_c$) and a spectrum bandwidth of BW Hz. We multiply the subcarriers $S(K)$ with a unique weighting function $A_i(K)$ ($1 \leq i \leq N$, $1 \leq K \leq n_c$) before ADC at each channel. $A_i(K)$ are composed by the channel characteristics of channel $i$. When the sampling rate of ADC is set to $1/N$ of the bandwidth of the signal to be sampled, i.e., BW/$N$ Sa/s, the aliasing of spectrum will happen according to the Nyquist sampling theorem. The aliasing mixes different parts of the spectrum by linear superposition and reduces the length of subcarriers. Denote the subcarriers at channel $i$ after the BW/$N$ Sa/s sampling as $R_i(k)$ ($1 \leq k \leq n_c/N$ which has a length of $n_c/N$. Then $R_i(k)$ can be expressed as the linear superposition of $A_i(K)S(K)$,

$$R_i(k) = \sum_{n=0}^{N-1} \mathrm{A}_i(k + nn_c/N)S(k + nn_c/N),$$
$$1 \leq k \leq n_c/N, \quad 1 \leq i \leq N. \tag{1}$$

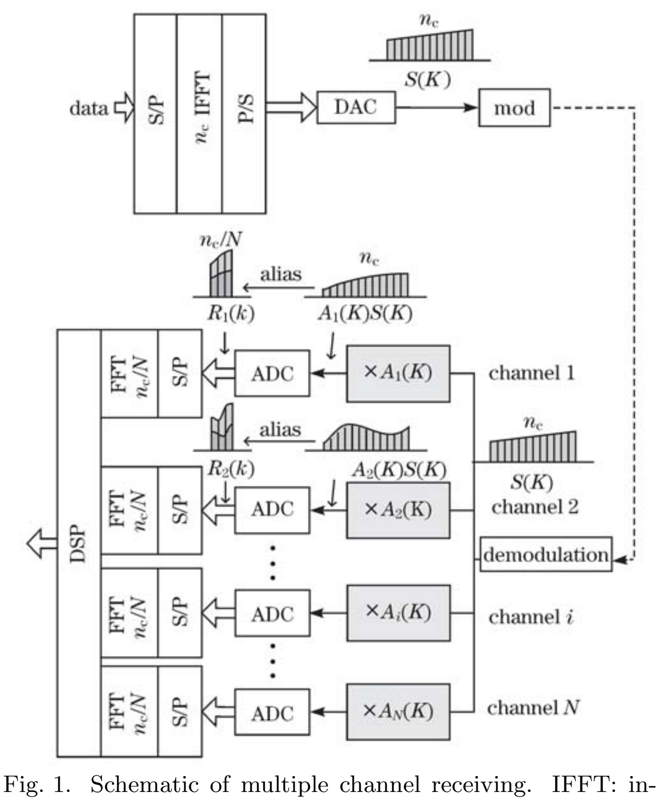


Fig. 1. Schematic of multiple channel receiving. IFFT: inverse FFT; DAC: DA converter; S/P: serial-to-parallel; P/S: parallel-to-serial.

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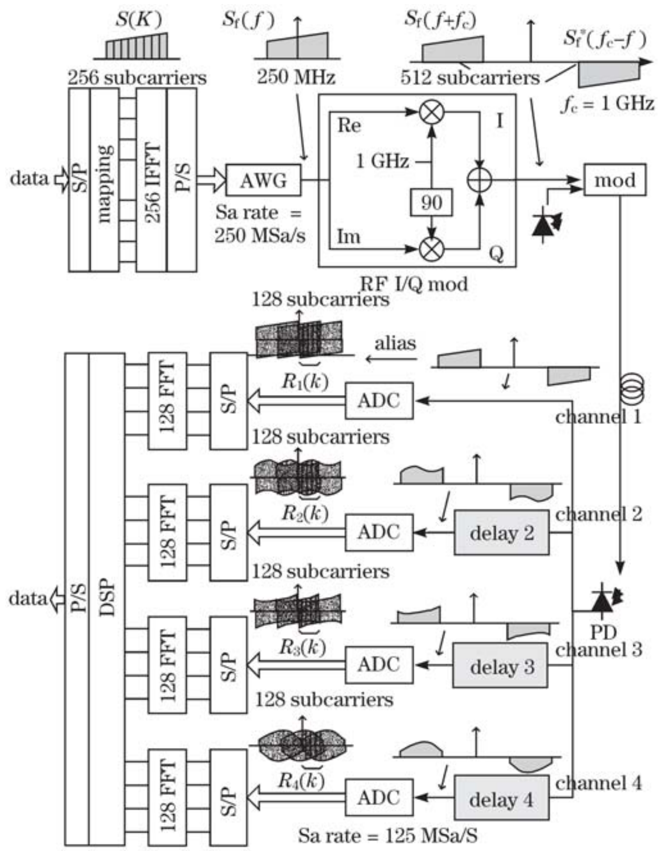


Fig. 2. Experimental setup of 4-channel receiving system. PD: photodiode; RF: radio frequency.

Therefore, we can get an equation set with $n_c$ independent equations for every subcarrier corresponding to the $N$ different receiving channels. $R_i(k)$ ($1\leq k \leq n_c/N$, $1\leq i\leq N$) is known as the output of the $n_c/N$-point FFT in channel $i$. Then the original OFDM subcarriers $S(K)$ ($1\leq K \leq n_c$) can be computed by solving Eq. (1).

As a back-to-back experiment, in Fig. 2, we demonstrate a scheme of a 4-channel receiver OOFDM system, wherein, a 250-MHz-bandwidth OFDM signal containing 256 subcarriers is generated by Agilent N8241A arbitrary waveform generator (AWG) and upconverted to an intermediate frequency at 1 GHz through in-phase/quadrature (I/Q) modulation which is complemented by Agilent E8267D vector signal generator. After electrical-to-optical and optical-to-electrical conversion through optical intensity modulation and direct detection, the detected signal is inputted into an Agilent DSA91204 to be sampled. Before the 125-MSa/s sampling, the signal in each channel is delayed by different time. The FFT size in each channel is 128 in comparison with that of 256 at the transmitter.

In our method, we avoid using the down-conversion block at the receiver for the heterodyne signal, but recover the base-band spectrum directly after low rate sampling and succeeding DSP. Without experiencing I/Q demodulation, the input to ADC in every channel will be the intermediate-frequency spectrum in a form of

$$S_{\mathrm{f}}(f+f_{\mathrm{c}})+S_{\mathrm{f}}^{*}(f_{\mathrm{c}}-f),$$

whose bandwidth is 500 MHz, twice that of base-band spectrum $S_{\mathrm{f}}(f)$. The band-pass sampling theorem demands the sampling rate should at least be 562.5 MSa/s to avoid an aliasing, while as the principle introduced above, with a 4-channel receiver, the sampling rate for every ADC can be reduced to 125 MSa/s in our experiment. Then, regarding to the 4×128 ouputs of the FFTs, we can get 512 equations as

$$\begin{aligned}&\mathrm{e}^{\mathrm{j}\omega_K t_i}S(k)+\mathrm{e}^{-\mathrm{j}\omega_{130-K}t_i}S^{*}(130-k)\\&+\mathrm{e}^{\mathrm{j}\omega_{128+K}t_i}S(128+k)+\mathrm{e}^{-\mathrm{j}\omega_{m(k)}t_i}S^{*}[m(k)]=R_i(k),\\&1\leq k\leq 128,\quad 1\leq i\leq 4,\end{aligned}\qquad(2)$$

where $\omega_K$ ($1\leq K \leq 256$) is the angular frequency of the $K$-th subcarrier at intermediate frequency; $t_i$ is the delay time in channel $i$, i.e., $t_1 = 0$, $t_2 = 12.5$ ns, $t_3 = 18.8$ ns, and $t_4 = 23.3$ ns in our experiment; $R_i(k)$ ($1 \leq k \leq 128$) is the output of the FFT block in channel $i$. $m(k)$ is a modulo operation, i.e., $m(k)$=(258−$k$) mod256.

The phase factors multiplied to the subcarriers play a role as the weighting function $A_i(K)$ ($1 \leq K \leq 256$) accounted in the principle section. Among the 512 equations, 256 are independent, e.g., in Eq. (2) the two equations are conjugate when $k$=65±$q$ ($1 \leq q \leq 63$). Hereto, 256 independent equations are built to solve the 256 unknown subcarriers, $S(K)$.

The constellation of the receiving data $S(K)$ is shown in Fig. 3(b). The constellation of conventionally receiving the same number of data points by full-bandwidth sampling in single channel is given in Fig. 3(a) for comparison. The well-remained structures of the constellations and error-free bits of 2-Mb data demonstrate our proposed scheme is feasible and effective.

In the experiment above, parameters are set as follows. Transmitter FFT size: 256, cycle prefix: 1/16, bit rate: 500 Mb/s, encoding: QPSK.

Excluding the joint points $S(1)$, $S(65)$, $S(129)$, and $S(193)$, Eq. (2) can be written in a matrix form of

$$\mathbf{A}\bar{\mathbf{S}}=\left(\mathbf{A}_1^{\mathrm{T}}\mathbf{A}_2^{\mathrm{T}}\mathbf{A}_3^{\mathrm{T}}\mathbf{A}_4^{\mathrm{T}}\right)^{\mathrm{T}}\bar{\mathbf{S}}=\mathbf{R},\qquad(3)$$

where $\overline{\mathbf{S}}=\Big[S(2)S(3)\cdots S(64)S^{*}(66)4\cdots S^{*}(128)S(130)\cdots S(192)S^{*}(194)\cdots S^{*}(256)\Big]^{\mathrm{T}}$, $\mathbf{R}=\Big[R_1(2)\cdots R_1(64)\; R_2(2)\cdots R_2(64)R_3(2)\cdots R_3(64)\;\; R_4(2)\cdots R_4(64)\Big]^{\mathrm{T}}$, the superscript T represents transpose operation of matrices, $\mathbf{A}_1-\mathbf{A}_4$ are the delay matrices expressed below,

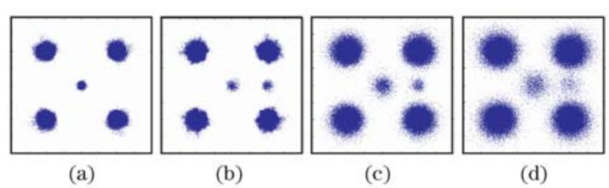


Fig. 3. Constellations of (a) conventional receiving and 4-channel receiving using delay lines with matrix condition numbers of (b) 1.54, (c) 8.12, and (d) 13.93.

$$\mathbf{A}_i=\begin{pmatrix} e^{j\omega_2 t_i} & & 0 & 0 & & e^{-j\omega_{128}t_i} & e^{j\omega_{130}t_i} & & 0 & 0 & & e^{-j\omega_{256}t_i} \\ & e^{j\omega_3 t_i} & & & \iddots & & & \ddots & & & \iddots & \\ & & \ddots & & \iddots & & & & \ddots & & \iddots & \\ 0 & & & e^{j\omega_{64}t_i} & e^{-j\omega_{66}t_i} & & 0 & 0 & & e^{j\omega_{192}t_i} & e^{-j\omega_{194}t_i} & 0 \end{pmatrix}_{i=1,2,3,4.}, \quad (4)$$

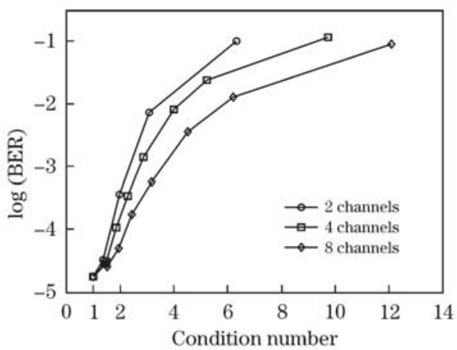


Fig. 4. Simulation results of BER impaired by the same noise versus condition number of Eq. (2) in different multi-channel systems, OSNR=11.5 dB.

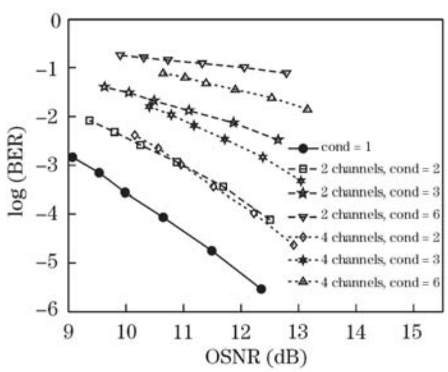


Fig. 5. Simulation results of OSNR sensitivity for different condition numbers in different multi-channel systems.

To compute $\mathbf{S}$, the inverse of the matrix $\mathbf{A}$ is needed. Thanks to the block-diagonal property of $\mathbf{A}$, its inverse remains block-diagonal and easy for computation. For every subcarrier, 4 complex multiplications and 3 complex additions are calculated, accordingly 252×4 complex multiplications and 252×3 complex additions are required for every OFDM symbol. This DSP cost is negligible since the significant speed of modern DSP devices.

This method can be extended to more channels with lower sampling rate ADCs. In an $N$-channel system with $n_c$ subcarriers, the matrix $\mathbf{A}$ is composed by $N\times N$ blocks which are diagonal matrices with a size of $(n_c/N-1)\times(n_c/N-1)$. In the succeeding DSP, $(n_c-N)\times N$ complex multiplications and $(n_c-N)\times(N-1)$ complex additions are required for every OFDM symbol.

It is worth noting that the delay time in every channel is arbitrary. The delay time can be realized by inserting a delay line without knowing its exact parameters which can be obtained by an inverse solution of Eq. (3) with training symbols. The only mathematical demand of these delays is $\det(\mathbf{A}) \neq 0$ to guarantee the independence of Eq. (2). However, the condition number[7] of matrix $\mathbf{A}$

$$\mathrm{cond}(\mathbf{A}) = ||\mathbf{A}^{-1}|| \cdot ||\mathbf{A}||, \quad (5)$$

which describes an upper bound of the sensitivity to perturbations in received subcarriers, is better to be small to provide a superior tolerance toward the noise added to the signal and a better recovery of the original signal. The condition number of matrix $\mathbf{A}$ in our experiment is 1.54. To illustrate the differences, we change the delay time to other two cases: 1) $t_1$=0, $t_2$=15.6 ns, $t_3$=17.3 ns, $t_4$=24.7 ns; 2) $t_1$=0, $t_2$=15.8 ns, $t_3$=16.2 ns, $t_4$=27.5 ns, which bring the matrix $\mathbf{A}$ condition numbers of 8.12 and 13.93, respectively. With a constant OSNR, deterioration of the constellation can be observed in Figs. 3(c) and (d). As the simulation results shown in Fig. 4, in a certain multi-channel receiving system, the bit error rate (BER) increases if the condition number does. The best case is that the condition number equals 1 in which the algorithm does not enlarge the noise at all, while the worst case is that the determinant of the matrix equals zero which brings an infinity of the condition number leading to an unsolvability of the subcarriers. System with more receiving channels has a better relationship between BER and condition number since the matrices are structured differently because of a mathematical problem which we will not discuss further here. The curves all start from a common point where the condition number equals 1. Figure 5 illustrates the OSNR sensitivity in systems with different delay lines and different numbers of receiving channels. When the condition number rises from 2 to 3, for example, the OSNR requirement increases by 1−2 dB in the same system. The curve at the bottom shows that all systems, no matter how many channels they have, perform the same sensitivity to OSNR if the condition number equals 1, and the multi-channel systems have an identical robustness as the conventional full-bandwidth receiving system.

In conclusion, we propose a cost-efficient method for high bit rate OOFDM receiving. The receiving is accomplished by multiple channels of low sampling rate ADCs and small size FFTs. Arbitrary time delay lines are inserted into the channels. An algorithm with mild computation to recover the original aliasing-free data is demonstrated and analyzed. A back-to-back experiment shows that the intermediate frequency OOFDM signal with bandwidth of 250 MHz can be retrieved directly by a 4-channel receiving system with ADCs with sampling rate of 125 MSa/s. The FFT size is also halved from 256 to 128. A superior array of delays can minimize the noise effect on the aliasing recovery. One of the disadvantages of our method is that it only reduces the sampling rate requirement of the ADCs but not the bandwidth requirement, which will be the focus in our future research.

This work was partially supported by the National Natural Science Foundation of China (Nos. 60736003 and 60932004), the National "863" Project of China (Nos. 2009AA01Z222, 2009AA01Z223, and 2009AA01Z256), the National "973" Project of China (No. 2006CB302805), and the Project iCHIP financed by Italian Ministry of Foreign Affairs.

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