Effects of group-velocity mismatch and cubic-quintic nonlinearity on cross-phase modulation instability in optical fibers

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The synthetic effects of group-velocity mismatch and cubic-quintic nonlinearity on cross-phase modulation induced modulation instability in loss single-mode optical fibers have been numerically investigated. The results show that the quintic nonlinearity plays a role similar to the case of neglecting the group-velocity mismatch in modifying the modulation instability, namely, the positive and negative quintic nonlinearities can still enhance and weaken the modulation instability, respectively. The group-velocity mismatch can considerably change the gain spectrum of modulation instability in terms of its shape, peak value, and position. In the normal dispersion regime, with the increase of the group-velocity mismatch parameter, the gain spectrum widens and then narrows, shifts to higher frequencies, and the peak value gets higher before approaching a saturable value. In the abnormal dispersion regime, two separated spectra may occur when the group-velocity mismatch is taken into account. With the increase of the group-velocity mismatch parameter, the peak value of the gain spectrum gets higher and shorter before tending to a saturable value for the first and second spectral regimes, respectively.

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It is well known that the interplay between the nonlinearity (self-phase modulation or cross-phase modulation) and dispersion effects inside optical fibers can result in a breakup of the continuous wave (CW) or quasi-CW optical wave into a train of ultra-short pulses. Since 1968, this so-called modulation instability has been extensively studied in various cases[1−20], for it can be used in many fields, such as optical switching[4], formation of chains of soliton like optical pulses[5,6], generation of super-continuum spectrum[7], measurement of nonlinear and chromatic dispersion parameters of optical fibers[8], etc.. On the other hand, however, previous researches have also shown that, whether self-phase or cross-phase modulation induced modulation instability may considerably degrade the performances of the optical fiber communication systems[9,10].

Self-phase modulation[11−13] and cross-phase modulation[14−16] induced modulation instability in optical fibers have both been extensively studied in the case of cubic nonlinearity of the refractive index. However, as previous work proposed[21], as high incident optical intensities or materials with very high nonlinear coefficients such as semiconductor doped glass optical fibers are considered, it is necessary to take high-order nonlinearities into consideration. The lowest high-order nonlinearity in optical fibers is quintic one. Thus, Tanev *et al.* investigated the solitary wave propagation and bistability behavior in optical fibers with cubic-quintic nonlinearity[21]. In addition, self-phase[18,19] and cross-phase modulation[3,20] induced modulation instability have also been investigated in the case of cubic-quintic nonlinearity. However, to our best knowledge, there are no reports on the synthetic effects of group-velocity mismatch and cubic-quintic nonlinearity on cross-phase modulation instability in optical fibers. Agrawal indicated that group-velocity mismatch can considerably influence cross-phase modulation instability[17].

When two optical pulses co-propagate in optical fibers, the linearised coupled nonlinear Schrödinger equation can be taken as[20]

$$\begin{aligned}&\frac{\partial a_j}{\partial z}+\frac{1}{V_{gj}}\frac{\partial a_j}{\partial t}+\frac{i}{2}\beta_{2j}\frac{\partial^2 a_j}{\partial t^2}\\&=i\gamma_{1j}P_j\exp\left(-\alpha_j z\right)\left(a_j+a_j^*\right)\\&+i2\gamma_{1j}\sqrt{P_1P_2}\exp\left(-\alpha_{3-j}z\right)\left(a_{3-j}+a_{3-j}^*\right)\\&+i2\gamma_{2j}P_j\exp\left(-\alpha_j z\right)\left[P_j\exp\left(-\alpha_j z\right)\right.\\&\quad\left.+3P_{3-j}\exp(-\alpha_{3-j}z)\right](a_j+a_j^*)\\&+i6\gamma_{2j}\sqrt{P_1P_2}\exp(-\alpha_{3-j}z)[P_j\exp(-\alpha_j z)\\&\quad+P_{3-j}\exp(-\alpha_{3-j}z)](a_{3-j}+a_{3-j}^*),\end{aligned}\qquad(1)$$

where a_j $(j=1,2)$ is the perturbation amplitude, V_{gj} the group velocity, β_{2j} the second-order group velocity dispersion coefficient, t the time coordinate, z the propagating distance, α_j the loss coefficient of the optical fiber, P_j the incident optical power, and γ_{1j} and γ_{2j} the third- and fifth-order nonlinear coefficients, respectively.

Adopting the well-known linear stability analysis[15,20], the dispersion relation can be obtained as

$$\left[\left(k-\frac{\Omega}{V_{g1}}\right)^2-f_1\right]\left[\left(k-\frac{\Omega}{V_{g2}}\right)^2-f_2\right]=C_{XPM},\qquad(2)$$

where k and Ω are the wave number and angle frequency of the modulation wave, respectively. It is obvious that this expression is completely the same as the case of cubic nonlinearity[17]. However, the definitions of the parameters f_1, f_2, and C_{XPM} are quite different, which read as

$$f_j = \frac{1}{2}\beta_{2j}\Omega^2\left[\frac{1}{2}\beta_{2j}\Omega^2 + 2\gamma_{1j}P'_j + 4\gamma_{2j}P'_j\left(P'_j + 3P'_{3-j}\right)\right], \tag{3}$$

and

$$C_{\mathrm{XPM}} = 4\beta_{21}\beta_{22}P'_1P'_2\Omega^4\left[\gamma_{11} + 3\gamma_{21}\left(P'_1 + P'_2\right)\right] \times\left[\gamma_{12} + 3\gamma_{22}\left(P'_1 + P'_2\right)\right], \tag{4}$$

where

$$P'_j = P_j\exp\left(-\alpha_j z\right). \tag{5}$$

Equation (2) is a fourth-degree polynomial with respect to wave number k. Under certain condition, if the roots of Eq. (2) make k become complex, modulation instability occurs and the corresponding power gain equals $2\mathrm{Im}(k)$. Im stands for the imaginary part. If either CW or sufficiently long pulses are input in the fiber, the group velocity mismatch can be neglected[14], i.e., $V_{\mathrm{g}1} \approx V_{\mathrm{g}2}$. In this case, the expressions for conditions as well as power gain of modulation instability can be obtained under $f_1f_2 < C_{\mathrm{XPM}}$,

$$g\left(\Omega\right) = 2\mathrm{Im}\left(k\right) = \sqrt{2}\{[(f_1+f_2)^2 + 4(C_{\mathrm{XPM}} - f_1f_2)]^{1/2} - (f_1+f_2)\}^{1/2}. \tag{6}$$

Generally speaking, however, taking into account the group velocity mismatch would be more accurate. Under this circumstance, it is difficult to obtain the analytical solution of Eq. (2) and the corresponding analytical expression for power gain coefficient of modulation instability. Accordingly, numerical processing is needed to solve Eq. (2) and discuss the modulation instability. To do so, one can set $\delta = V_{\mathrm{g}1}^{-1} - V_{\mathrm{g}2}^{-1}$ and $K = k - \frac{\Omega}{V_{\mathrm{g}2}}$, where δ is the group velocity mismatch parameter. After substituting these assumptions in Eq. (2), one can obtain

$$\left\{(K - \delta\Omega)^2 - f_1\right\}\left\{K^2 - f_2\right\} = C_{\mathrm{XPM}}. \tag{7}$$

For the relation $\mathrm{Im}(k) = \mathrm{Im}(K)$ holds, the power gain of modulation instability, which equals $2\mathrm{Im}(K)$, can be obtained by numerical solving Eq. (7). Our numerical simulation indicates that four roots of Eq. (7) which correspond to the four gain spectra of modulation instability actually consist of two couples of conjugate roots. But only the spectrum, which has the most number of the separated spectral regions, is adopted in the following discussion.

If the wavelength difference of the two optical waves is not too big, one can assume that $\gamma_{11} \approx \gamma_{12} = \gamma_1$, $\gamma_{21} \approx \gamma_{22} = \gamma_2$, $\alpha_1 \approx \alpha_2 = \alpha$. Further, when the third-order group velocity dispersion coefficient is small enough and negligible, one can also set $\beta_{21} \approx \beta_{22} = \beta_2$ as Ref. [17]. The common parameters used in the following calculations are $\gamma_1 = 1$ W^{-1}·km^{-1}, $z = 1$ km, $P_1 = P_2 = 500$ W, $\alpha = 0.2$ dB/km. And the concrete value of parameter γ_2 is set by consulting Ref. [18].

Figure 1 shows the gain spectra of cross-phase modulation induced modulation instability in the normal dispersion regime for different group-velocity mismatch parameters when the quintic nonlinearity coefficients are (a) $\gamma_2 = 6\times10^{-5}$ W^{-2}·km^{-1}, (b) $\gamma_2 = 0$, and (c) $\gamma_2 = -6\times10^{-5}$ W^{-2}·km^{-1}, respectively. $\beta_2 = 20$ ps^2/km. Comparing Figs. 1(a) with (b) and (c), one can realize that when the group-velocity mismatch parameter and other parameters are the same, the positive quintic nonlinearity makes the spectral width wider and the peak value of the spectra higher. That is to say, modulation instability can occur in a wider frequency region and the perturbation optical field will grow faster in the case of positive quintic nonlinearity. While the negative quintic nonlinearity decreases the spectral width as well as the peak value of the spectra. In other words, in case of the group-velocity mismatch, the positive and negative quintic nonlinearities still enhance and weaken the modulation instability, respectively. This result accords with

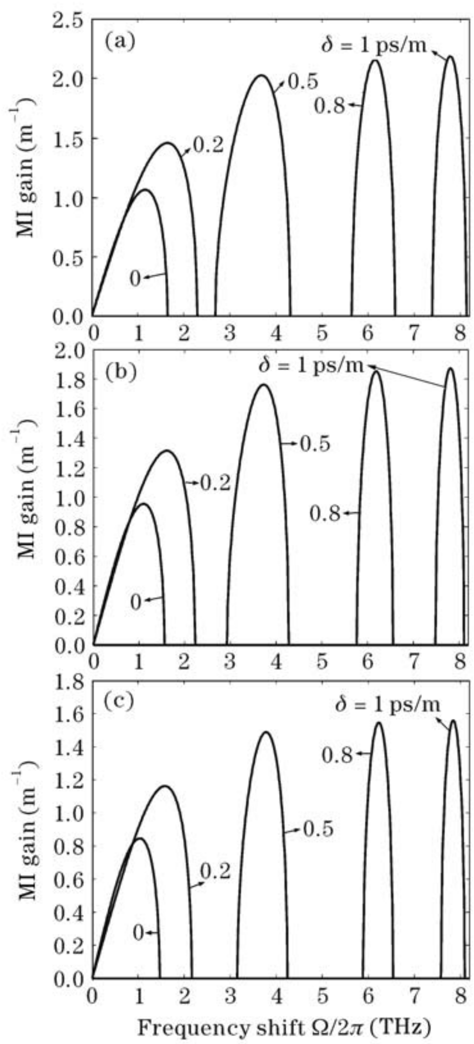

Fig. 1. Gain spectra of cross-phase modulation induced modulation instability (MI) in the normal dispersion regime for different group-velocity mismatch parameters when the quintic nonlinearity coefficients are (a) $\gamma_2 = 6\times10^{-5}$ W^{-2}·km^{-1}, (b) $\gamma_2 = 0$, and (c) $\gamma_2 = -6\times10^{-5}$ W^{-2}·km^{-1}, respectively. $\beta_2 = 20$ ps^2/km.

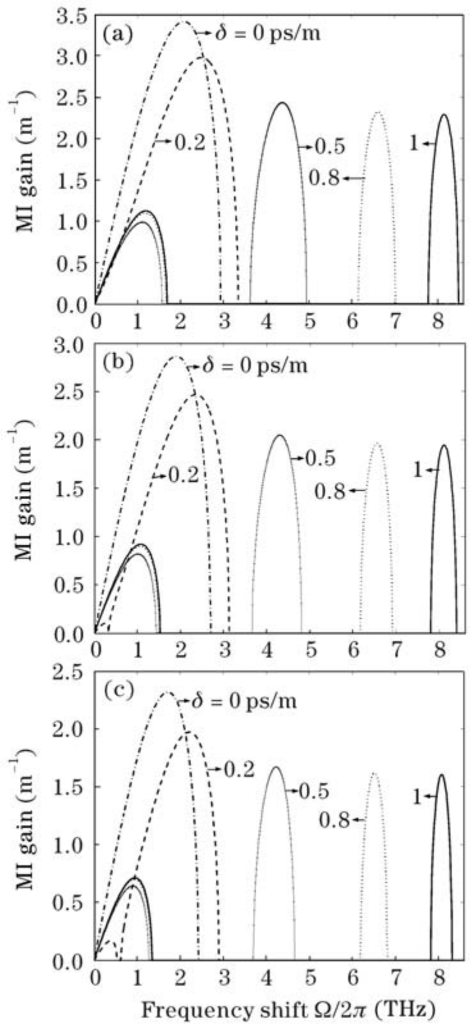

Fig. 2. Gain spectra of cross-phase modulation induced modulation instability (MI) in the abnormal dispersion regime for different group-velocity mismatch parameters when the quintic nonlinearity coefficients are (a) $\gamma_2 = 6 \times 10^{-5}$ $\mathrm{W^{-2} \cdot km^{-1}}$, (b) $\gamma_2 = 0$, and (c) $\gamma_2 = -6 \times 10^{-5}$ $\mathrm{W^{-2} \cdot km^{-1}}$, respectively. $\beta_2 = -20$ $\mathrm{ps^2/km}$.

that of Ref. [20]. It can also be seen that, the group-velocity mismatch can considerably alter the gain spectrum in terms of its shape, peak value, and position. Concretely, when the other conditions are the same, with the increase of the group-velocity mismatch parameter, the gain spectrum widens and then narrrows, shifts to higher frequencies, and the peak value gets higher before tending to a saturable value. When the quintic nonlinearity is neglected, our simulation results here are completely in agreement with that of Ref. [17] if other parameters are the same.

Figure 2 shows the gain spectra of modulation instability in the abnormal dispersion regime for different group-velocity mismatch parameters when the quintic nonlinearity coefficients are (a) $\gamma_2 = 6 \times 10^{-5}$ $\mathrm{W^{-2} \cdot km^{-1}}$, (b) $\gamma_2 = 0$, and (c) $\gamma_2 = -6 \times 10^{-5}$ $\mathrm{W^{-2} \cdot km^{-1}}$, respectively. $\beta_2 = -20$ $\mathrm{ps^2 \cdot /km}$. In comparison, the most obvious characteristic of the gain spectra in the abnormal dispersion regime is that two separated spectra may occur in the case of group-velocity mismatch, which is quite different from the case of neglecting the group-velocity mismatch where only one spectrum exists[20]. The first spectrum is near the center wavelength and has smaller peak value gain. The second one is far away from the center wavelength and has higher peak value gain. When the group-velocity mismatch parameter is small or zero, there exists only the first spectrum. With the increase of the group-velocity mismatch parameter, the peak value of the gain spectrum gets higher and shorter before tending to a saturable value for the first and second spectral regimes, respectively. Similarly, the positive and negative quintic nonlinearities also enhance and weaken the modulation instability in this case, respectively.

According to the extended coupled nonlinear Schrödinger equations in loss single-mode optical fibers with cubic-quintic nonlinearity, for two optical waves of different wavelengths with the same polarization but with nonoverlapping spectra, the dispersion relation for wave number k of the modulation wave is numerically solved. The synthetic effects of group-velocity mismatch and cubic-quintic nonlinearity on modulation instability induced by cross-phase modulation have been numerically investigated. The results show that the quintic nonlinearity plays a similar role in the case of neglecting the group-velocity mismatch in modifying the modulation instability, namely, the positive and negative quintic nonlinearities can also enhance and weaken the modulation instability, respectively. However, the group-velocity mismatch can considerably modify the gain spectrum of modulation instability in terms of its shape, peak value, and position. In the normal dispersion regime, with the increase of the group-velocity mismatch parameter, the gain spectrum narrows, shifts to higher frequencies, and the peak value gets higher before approaching a saturable value. In the abnormal dispersion regime, two separated spectra may occur in the case of group-velocity mismatch, which is quite different from that of Ref. [20] where only one spectrum exists. With the increase of the group-velocity mismatch parameter, the peak value of the gain spectrum gets higher and shorter before tending to a saturable value for the first and the second spectral regimes, respectively.

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